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The Relationship of Direct Price Flexibilities to Direct Price Elasticities

JAMES P. HOUCK

In agricultural economics research, it is frequently easier to estimate direct and cross price flexibilities rather than price elasticities. However, elasticity estimates may be needed or wanted. The paper shows that, under rather general conditions, the reciprocal of the direct price flexibility is the lower absolute limit of the direct price elasticity. The departure of the true price elasticity from the flexibility reciprocal depends upon the strength of the cross effects of substitution and complementarity with other commodities.

FOR ONE reason or another, economists are often motivated to measure price elasticities of demand. In agricultural economics, however, it is frequently more appropriate to estimate direct and cross price flexibilities instead.[1] This is because many agricultural production processes are such that market supplies of related commodities are determined largely in advance of current prices. As a result, the ordinary least squares procedure is often applied to time series data with a particular commodity price expressed as the dependent variable. The independent variables, typically, are the fixed supplies of that and related commodities along with other "shift" variables. When the observations are expressed as logarithms, the estimated coefficients may be interpreted as direct and cross price flexibilities at the specified marketing level.

An economist wishing to discuss the direct price elasticity of demand for this commodity is apt to assert that the reciprocal of his computed direct price flexibility is a good estimate of the price elasticity. Apt, that is, unless he has investigated the warnings against this procedure sprinkled through recent agricultural economics literature. For instance in 1956, Meinken, Rojko, and King wrote that:

> . . . the reciprocal of the "price flexibility" . . . equals the price elasticity . . . *only* if β_{12} and β_{21} [cross flexibilities] are zero.[2]

[1] Price flexibility is the percentage change in the price of a commodity associated with a 1 percent increase in the quantity demanded of that commodity or a related variable, all else remaining constant.

[2] Meinken, Rojko, and King, "Measurement of Substitution in Demand from Time Series Data—A Synthesis of Three Approaches," *J. Farm Econ.*, Vol. 38, August 1956, p. 734.

JAMES P. HOUCK *is visiting assistant professor of business research, Harvard Business School.*

Harlow in 1962 remarked:

> If the effects of other goods are taken into account, the price elasticity is greater than that obtained by taking the reciprocal of price flexibility.[3]

Waugh recently wrote:

> The reciprocals of price flexibilities are often taken to represent elasticities of demand. . . . I prefer to use flexibilities themselves rather than their reciprocals. If, for any reason, the elasticity of demand is wanted, I would prefer to use . . . regression equations, using quantities as the dependent variables. . . .[4]

The major implications of these and a number of other scattered observations are:

1. The reciprocal of the direct price flexibility is not in general the same as the direct price elasticity.
2. The reciprocal of the price flexibility is absolutely less than the true elasticity if there are discernible cross effects with other commodities.

Although the relationships among estimated demand and supply coefficients have been examined at length, the link between the direct price flexibility and elasticity of demand has not been discussed explicitly in the literature.[5] Although often mentioned in passing, this particular relationship remains a source of confusion. In order to clarify it, only a little matrix algebra and some economic theory are needed. It is shown here that, under rather general conditions, the reciprocal of the direct price flexibility (often estimated in econometric work) is the lower absolute limit of the corresponding direct price elasticity. The difference between the two depends upon the strength of the cross effects of commodity substitution and, if relevant, complementarity.[6]

Consider a square matrix E of direct and cross price elasticities of demand for an exhaustive set of n commodities at a specified marketing

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[3] A. A. Harlow, *Factors Affecting the Price and Supply of Hogs*, USDA, Tech. Bul. 1274, 1962, p. 41.

[4] F. V. Waugh, *Demand and Price Analysis: Some Examples from Agriculture*, USDA Tech. Bul. 1316, 1964, pp. 29-30. Logical relationships among price flexibilities are developed in Appendix 3, pp. 80-85.

[5] Perhaps the clearest discussion of the relationships among estimated elasticities in models with both supply and demand functions is R. C. Buse, "Total Elasticities—A Predictive Device," *J. Farm Econ.*, Vol. 40, November 1958, pp. 881-891. E. W. Learn illustrates similar properties among various price elasticities in a simultaneous demand model in "Estimating Demand for Livestock Products," *J. Farm Econ.*, Vol. 38, December 1956, pp. 490-491.

[6] These relations may be obscured in real situations if changes in stocks or net exports are important; see R. J. Foote, *Analytical Tools for Studying Demand and Price Structures*, Agr. Hndbk. 146, 1958, p. 82.

level. Assume that E is the usual type of demand elasticity matrix in a general equilibrium model with direct elasticities down the diagonal and cross elasticities arranged around the diagonal in symmetric positions.[7] That is, the elasticity at the intersection of the i^{th} row and the j^{th} column, e_{ij}, is the percent change in the quantity demanded of i associated with an isolated 1 percent increase in the price of j. Similarly the elasticity in the ji^{th} cell, e_{ji}, is the percentage quantity change in j associated with an isolated 1 percent increase in the price of i. Let F be the corresponding matrix of price flexibilities for the n commodities in which f_{ij} is the percentage price change in i associated with an isolated 1 percent increase in the quantity of j. It has been shown that:

$$F = E^{-1} \text{ and } FE = I$$

where I is an identity matrix with n ones on the main diagonal and zeros elsewhere.[8] Thus, the sum of the inner products of the i^{th} row of F and the i^{th} column of E is equal to one. For instance, consider the first row and column of F and E respectively:

$$f_{11}e_{11} + f_{12}e_{21} + f_{13}e_{31} \cdots f_{1n}e_{n1} = 1$$

or

$$f_{11}e_{11} + \sum_{k=2}^{n} f_{1k}e_{k1} = 1.$$

The first product $f_{11}e_{11}$ will be positive and equal to or greater than one. This follows because each of the other terms, in general, will be negative or zero no matter whether the individual cross effects stem from substitution or complementarity. For example, if commodity 2 is a substitute for commodity 1, then f_{12} will be negative and its counterpart in E, e_{12}, will be positive. But, because of the symmetry relation among demand elasticities, e_{21} can also be expected to be positive, making $f_{12}e_{21}$ (the second term of the above equation) negative.[9] Should commodity 3 be complementary with commodity 1, then f_{13} will be positive and e_{13} negative. Therefore, e_{31} can be expected to be negative and $f_{13}e_{31}$ (the third term of the equation) also negative. If two or more commodities

[7] Examples of matrices of this type at retail and farm levels are presented and discussed in detail by G. E. Brandow, *Interrelations Among Demands for Farm Products and Implications for Control of Market Supply*, Pennsylvania Agr. Expt. Sta. Bul. 680, 1961, pp. 17 and 59.

[8] R. J. Foote, *op. cit.*, pp. 90-94.

[9] Wold and Jureen, *Demand Analysis*, Wiley, 1953, p. 112. This symmetry relation will produce similar signs for related coefficients in all but very unusual or extreme conditions. Brandow's work (*op. cit.*) suggests that demand coefficients for agricultural products are reasonably well-behaved.

are independent, then the inner product terms will be zero. Since all terms of the equation but the first are negative or zero:

$$f_{11}e_{11} \geq 1$$

or

$$|e_{11}| \geq \left|\frac{1}{f_{11}}\right|$$

and, in general

$$|e_{ii}| \geq \left|\frac{1}{f_{ii}}\right| .$$

The reciprocal of the direct price flexibility forms the *lower limit,* in absolute terms, of the direct price elasticity.[10] The stronger the cross effects of substitution and complementarity, the further e_{ii} will depart from $1/f_{ii}$.[11] If all cross effects are zero, then e_{ii} will equal $1/f_{ii}$.

If an economist finds significant cross flexibility estimates with other commodities in his equation, then the reciprocal of the direct flexibility is surely not a reliable elasticity estimate. However, the knowledge that it is an estimate of the lower bound of the elasticity may be useful for many purposes.[12] As an example, suppose that the government increases the price support for some commodity above prevailing open market prices by direct purchases. The reciprocal of the estimated direct price flexibility for this commodity would be an indication of the *minimum* percentage decrease in quantity demanded at the specified level of market due to each 1 percent increase in the support level over the market price. This, along with cross flexibility estimates and production forecasts for the related commodity group, would be valuable information in planning the strategy of government purchases, storage, and ultimate disposition of this commodity. In addition, the investigator can get some idea of the strength of the departure of the true elasticity from the flexibility reciprocal by examining the size and significance of the estimated cross flexibilities. An accurate measure of this difference depends on elasticity estimates for other commodities which may or may not be available.

[10] The inner product sums of F and E which yield the nondiagonal zeros do not provide readily useful relationships among the coefficients when only flexibilities have been estimated.

[11] It is equally true that the reciprocal of the direct elasticity is the lower absolute limit of the direct flexibility. For empirical comparisons of direct elasticities and flexibilities at the farm level, see Tables 12 and 13 in Brandow, *op. cit.,* pp. 59 and 65.

[12] The varied and difficult problems of interpreting and using elasticity estimates are discussed in R. J. Foote, *op. cit.,* pp. 82-83; and L. V. Manderscheid, "Some Observations on Interpreting Measured Demand Elasticities," *J. Farm Econ.,* Vol. 46, February 1964, pp. 128-137.