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STATISTICAL EXAMINATION OF RESERVOIR-INDUCED SEISMICITY

BY GREGORY B. BAECHER AND RALPH L. KEENEY

ABSTRACT

The two purposes of this work are to better understand the correlates of reservoir-induced seismicity (RIS) and to develop a model for predicting RIS from reservoir characteristics. Data from 29 reservoirs associated with RIS and 205 reservoirs not associated are analyzed using statistical discriminant analysis. These analyses show significant correlations between RIS and reservoir depth and between RIS and reservoir volume, but lesser correlations of RIS with stress and with geology. Data on fault activity are too few to allow inferences of correlation.

A prediction model based on discriminant analysis and calibrated by the data provides a rough estimate of the probability of RIS for specific sites. This model does not allow precise estimates, but it does distinguish between probabilities of RIS in the range of, say 30 per cent from those of 5 per cent. The base-rate frequency of RIS, knowing nothing more than that reservoir depth exceeds 92 m (deep and very deep reservoirs), is about 14 per cent. Given the most favorable set of reservoir attributes, the present model would reduce this probability to about 3 per cent. Given the least favorable, the model would increase this probability to almost 70 per cent. The standard deviation of the larger estimates is on the order of 12 per cent (absolute).

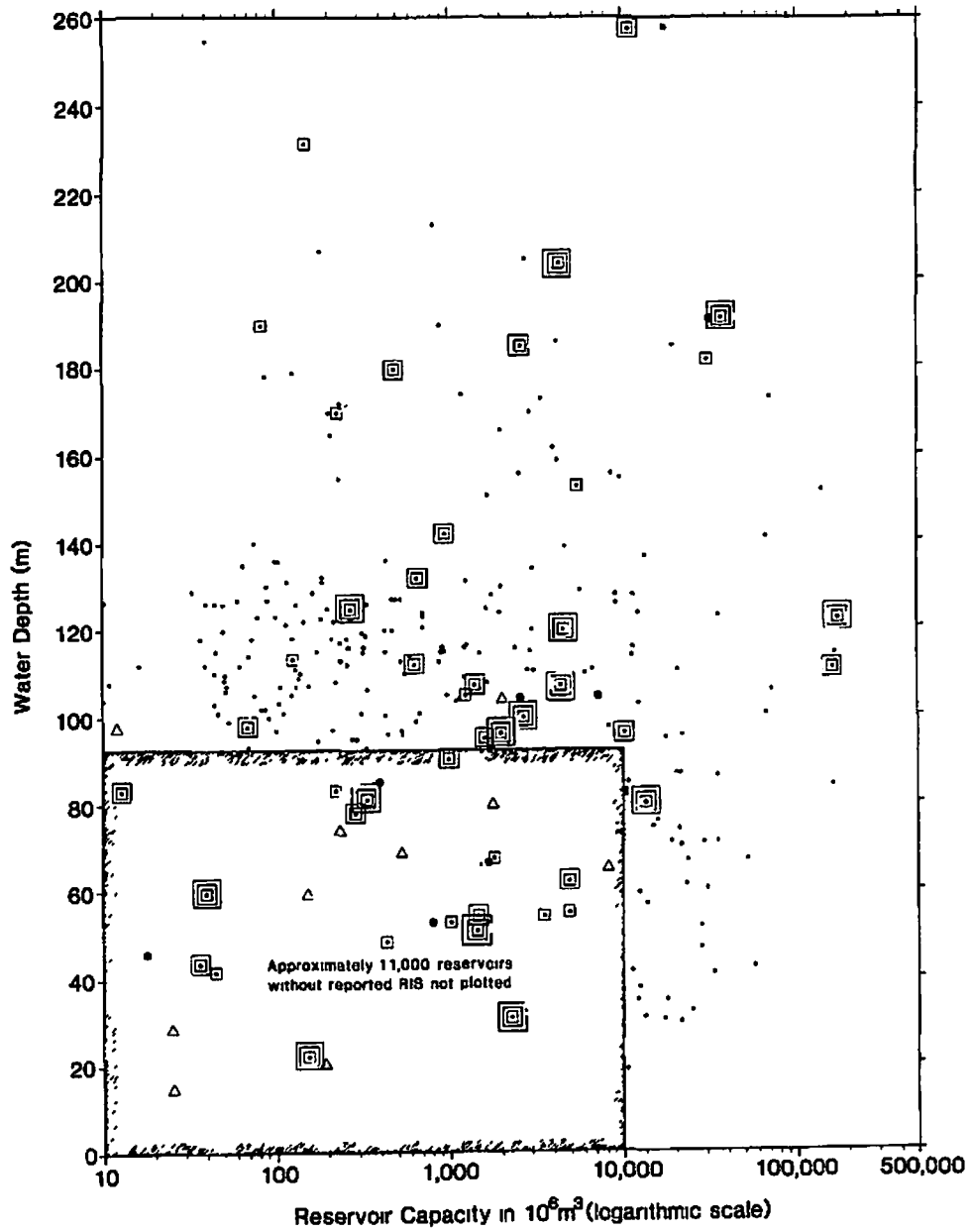
1. INTRODUCTION

There has been increasing interest over the past decade in the occurrence of reservoir-induced seismicity (RIS) (e.g., Milne, 1976), and during the past few years, a large data base has been gathered (Packer *et al.*, 1979). This data base has been carefully assembled to identify those reservoirs generally agreed to have induced seismic activity, and to specify characteristics of those reservoirs based on design documents, individual inspection, or published sources to associate geological and engineering characteristics with them. To the date of the present analysis, this compilation has required several person-years of effort.

Professional interest has focused recently on attempts to understand RIS through modeling. That is, by reasoning from first principles, geomechanics has been used to explain RIS and in the future may be used to predict its occurrence. A complementary way to study the phenomenon is empirical, by analyzing the historical record. Few attempts in this second direction have been reported that systematically evaluate existing data and address the significance of inferences drawn from the data. The present study was an attempt to do so. Specifically, the objectives of the study were

1. to better understand the phenomena of RIS through analysis of the historical record, and
 2. to develop a model to predict the likelihood of RIS occurrence for specific sites.
- No one approach to analyzing RIS provides a definitive answer. The intent of the present work is to complement other approaches and to provide one more step toward a rational procedure for dealing with a difficult problem. As with any empirical analysis, conclusions reported here are based on the current historical data base.

This paper is organized in four subsequent sections. Section 2 describes the data base. Section 3 presents the data analysis. Section 4 presents three preliminary models to predict RIS, and section 5 discusses the quality of the data.



Note: The following reservoirs were not plotted because of insufficient data: Kinarsam, Sharavathi
 • - Nurek (USSR) depth is in excess of 285 m

EXPLANATION

- Deep and/or very large reservoir
- ◻ Accepted case of RIS, maximum magnitude ≥ 5
- ◻ Accepted case of RIS, maximum magnitude 3-5
- ◻ Accepted case of RIS, maximum magnitude ≤ 3
- △ Questionable case of RIS
- Not RIS

FIG. 1. Relationship of depth and volume and reported cases of RIS

2. THE DATA BASE

There are approximately 11,000 reservoirs in the world. Because of the effort necessary to collect data on 11,000 reservoirs, the study focused on three subsets: deep and very deep reservoirs; very large reservoirs; and reservoirs with reported cases of RIS (Figure 1). The reservoirs were characterized with respect to five attributes: depth; volume; stress state; presence of active faulting; and geology. These attributes were chosen because they are thought to correlate with RIS, and because data are usually available to classify a particular reservoir. Other attributes, like fluid pressures at depth, were not used because they cannot be evaluated from commonly existing records.

Attribute definitions are shown in Table 1. Depth and volume are self-explanatory. Stress refers to the orientation of principal stresses. Extensional means the minor principal stress is vertical; compressional means the major principal stress is vertical; shear means the intermediate principal stress is vertical. Faulting refers to whether active faulting was or was not observed in the vicinity of the reservoir prior to reported RIS. Geology refers to predominant local formations, sedimentary, igneous, or metamorphic. The procedure for classifying individual dams and an extended discussion of the documentary support are given by Packer *et al.* (1979).

TABLE 1
DEFINITION FOR RESERVOIR ATTRIBUTE STATES

Attribute	State			
	1	2	3	4
Depth	d_1 very deep (over 150 m)	d_2 deep (92 to 150 m)	d_3 shallow (less than 92 m)	d_4 not known
Volume	v_1 very large (over 100 × 10 ⁶ m ³)	v_2 large (12 to 100 × 10 ⁶ m ³)	v_3 small (less than 12 × 10 ⁶ m ³)	v_4 not known
Stress State	s_1 : extensional	s_2 compressional	s_3 shear	s_4 not known
Fault Activity	f_1 active fault present	f_2 no active faults present	f_3 not known	
Geology	g_1 = sedimentary	g_2 : metamorphic	g_3 : igneous	g_4 : not known

The abbreviations used in the tables are: d , depth; v , volume; s , stress state, f , fault activity, and g , geology.

Classification of seismic events as reservoir induced was made only after study of individual records. Because these records are at times incomplete, and at times conflicting, each classification was reviewed by a project team of geologists, seismologists, and geotechnical engineers. The reservoirs classified in this study as having induced seismicity and used in subsequent statistical analyses, are listed in Table 2. Reservoir-induced events of Richter magnitude 3 or greater were considered macroseismic; those less than 3 were considered microseismic.

3. ANALYSIS OF THE DATA

The data were first examined to determine relationships between single attribute states and the occurrence of RIS. In particular, two data sets were studied: the set of reservoirs that was deep, very deep, or very large; and the set of reservoirs that was only deep or very deep. The first set contained 29 instances of RIS and 205 instances of no RIS; the second contained 28 instances of RIS and 172 instances of no RIS. Numbers (and frequencies) of occurrence of attribute states and sampling

variances are shown in Table 3 for these two data sets. For instance, in the first data set 10 of the 29 reservoirs with RIS were categorized as very deep (34 per cent).

For the set of 234 reservoirs that are either deep, very deep, or very large, the frequency of RIS given no specific knowledge of the reservoir itself is 0.12. This is simply the number of RIS cases (29) divided by the total number of such reservoirs. For the second data set, the corresponding frequency is 0.14.

For each attribute taken individually, the frequencies of various states are different for the RIS and non-RIS reservoirs. Assuming the present data are a sample of all possible reservoirs, the best (maximum likelihood) estimates of the frequencies for all possible reservoirs are given in Table 3. Let " p " be the frequency of a

TABLE 2
RESERVOIRS CLASSIFIED AS HAVING INDUCED SEISMICITY THAT
WERE USED IN THE STATISTICAL ANALYSIS

1. Akosombo Main, Lake Volta	Ghana
2. Almendra	Spain
3. Benmore	New Zealand
4. Blowering	Australia
5. Canelles	Spain
6. Contra, Lake Vogorno	Switzerland
7. Emosson	Switzerland
8. Eucumbene	Australia
9. Grancarevo	Yugoslavia
10. Hoover, Lake Mead	USA
11. Jocassee	USA
12. Kariba	Zambia/Rhodesia
13. Keban	Turkey
14. Koyna, Shivaji Sagar Lake	India
15. Kremasta	Greece
16. Kurobe	Japan
17. La Cohulla	Spain
18. Manicougan 3	Canada
19. Monteynard	France
20. Nurek	USSR
21. Oroville	USA
22. Pieve di Cadore	Italy
23. Schlegeis	Austria
24. Shasta	USA
25. Talbingo	Australia
26. Vajont	Italy
27. Vouglans	France
28. Warragamba, Lake Burragorang	Australia
29. Xinfengjiang	China

particular attribute state for individual reservoirs. Because of statistical fluctuation in data, estimates of the parameter p may vary from one data set to the next. Adopting a Bernoulli model, the sampling variance of these fluctuations is $p(1-p)/n$, where " n " is the number of data in the set. Thus, for example, because 10 of the 29 RIS reservoirs were very deep, the best estimate of p , denoted \hat{p} , is 0.34, and hence, of $1 - \hat{p}$, is 0.66. The variance $V[\hat{p}]$ associated with \hat{p} is $(0.34)(0.66)/29 = 0.0077$ so that the standard deviation, which is the square root of the variance, is 0.088. Strictly, this means, that if 34 per cent of all reservoirs inducing seismicity were very deep, then the expected frequency in our sample would also be 34 per cent, but this might vary with a standard deviation of 0.088. More roughly, if data

TABLE 3
RELATIONSHIPS OF ATTRIBUTE STATES TO RIS

	No of Reservoirs	State		
		1	2	3
(a) Likelihoods of Attribute States for RIS and Non-RIS—Independent Case*				
RIS Data				
Depth	29	10 (0.34)	18 (0.62)	1 (0.04)
Volume	29	7 (0.24)	11 (0.38)	11 (0.38)
Stress State	29	4 (0.14)	18 (0.62)	7 (0.24)
Fault Activity	7	7 (1.00)	0 (0.00)	
Geology	28	13 (0.46)	8 (0.29)	7 (0.25)
Non-RIS Data				
Depth	204	27 (0.13)	144 (0.71)	33 (0.16)
Volume	205	52 (0.25)	36 (0.18)	117 (0.57)
Stress State	203	34 (0.17)	138 (0.68)	31 (0.15)
Fault Activity	6	4 (0.67)	2 (0.33)	
Geology	165	57 (0.35)	64 (0.39)	44 (0.26)
(b) Likelihoods of Attribute States for RIS and Non-RIS—Independent Case†				
RIS Data				
Depth	28	10 (0.36)	18 (0.64)	0
Volume	28	6 (0.22)	11 (0.39)	11 (0.39)
Stress State	28	4 (0.14)	18 (0.64)	6 (0.22)
Fault Activity	6	6 (1.0)	0 (0.00)	
Geology	27	13 (0.48)	8 (0.30)	6 (0.22)
Non-RIS Data				
Depth	171	27 (0.16)	144 (0.84)	0
Volume	171	18 (0.11)	36 (0.21)	117 (0.68)
Stress State	171	33 (0.19)	109 (0.64)	29 (0.17)
Fault Activity	6	4 (0.67)	2 (0.33)	
Geology	143	44 (0.31)	60 (0.42)	39 (0.27)
(c) Sampling Variance of Attribute Likelihoods—Independent Case‡				
RIS Data ($\times 10^{-3}$)				
Depth		7.7 (8.2)	8.1 (8.2)	1.0
Volume		6.3 (5.9)	8.1 (8.5)	8.1 (8.5)
Stress State		4.2 (4.3)	8.1 (8.2)	6.3 (6.1)
Fault Activity		—	—	
Geology		8.9 (9.2)	7.4 (7.8)	6.7 (6.4)
Non-RIS Data ($\times 10^{-3}$)				
Depth		0.56 (0.78)	1.0 (0.78)	0.66
Volume		0.92 (0.55)	0.72 (0.97)	1.2 (1.3)
Stress State		0.70 (0.90)	1.1 (1.3)	0.63 (0.85)
Fault Activity		37.0 (37.0)	37.0 (37.0)	—
Geology		1.4 (1.5)	1.4 (1.7)	1.2 (1.4)

* This section summarizes the deep, very deep, or very large reservoir data. The frequency of reservoirs having an attribute state is in parentheses.

† This section summarizes deep or very deep reservoir data only. The frequency of reservoirs having an attribute state is in parentheses.

‡ Numbers in parentheses in this section refer to deep or very deep data set. The others refer to the deep, very deep, or very large data set.

were available on a very large number of reservoirs, there is approximately an 85 per cent (for large sample sizes, the sampling distribution of \hat{p} approaches normality, and therefore, probabilities can be taken from tables of the normal distribution) chance that the frequency of very deep reservoirs among the RIS set would be between 0.34 minus one standard deviation and 0.34 plus one standard deviation (or from 0.25 to 0.43).

Sampling variances for the data of Table 3, a and b, are shown in c. The variances are uniformly smaller for the non-RIS set. This simply reflects the difference in sample sizes. Because the present analysis is based on techniques of classical estimation, empty data cells yield estimates where \hat{p} equals zero. For example, of

TABLE 4
DATA TO EXAMINE ATTRIBUTE CORRELATIONS*

RIS			Non-RIS				
d_1	4	3	3	d_1	11	11	5
d_2	7	8	3	d_2	106	25	13
d_3	—	—	1	d_3	—	—	33
	v_3	v_2	v_1		v_3	v_2	v_1
d_1	3	5	2	d_1	4	17	5
d_2	3	13	2	d_2	25	91	28
d_3	1	0	0	d_3	2	29	1
	s_3	s_2	s_1		s_3	s_2	s_1
v_1	2	1	4	v_1	11	9	17
v_2	3	4	4	v_2	5	11	15
v_3	2	3	5	v_3	28	44	25
	g_3	g_2	g_1		g_3	g_2	g_1
s_3	2	3	2	s_3	22	4	5
s_2	9	6	3	s_2	79	19	40
s_1	0	2	2	s_1	16	13	5
	v_3	v_2	v_1		v_3	v_2	v_1
s_3	3	2	2	s_3	13	8	4
s_2	4	4	9	s_2	25	43	43
s_1	0	2	2	s_1	5	11	9
	g_3	g_2	g_1		g_3	g_2	g_1
d_1	2	5	3	d_1	2	12	9
d_2	4	3	10	d_2	37	48	35
d_3	1	0	0	d_3	5	4	13
	g_3	g_2	g_1		g_3	g_2	g_1

* Cell frequencies show numbers of reservoirs having paired combinations of attributes.

the RIS sites having data on fault activity, all seven were active (f_1). Therefore, $\hat{p}(f_2 | \text{RIS}) = 0$, and $V[\hat{p}] = 0$. This is an aberration of the statistical techniques used. Because of the small sample size, one should be most careful in drawing conclusions from this circumstance.

Correlations among attributes. Statistical procedures were used to test whether apparent pair-wise correlations among attributes were significant. For lack of data, faulting was excluded; thus, correlations of six possible pairs were tested for both the RIS and non-RIS reservoirs (Table 4).

Table 5 illustrates the procedure using depth and volume for the 28 deep and very deep RIS reservoirs, since data were not available on all shallow dams. Of these, 18

are deep and 10 are very deep. Consequently, the estimated frequency of very deep dams is 10/28, and of deep dams 18/28. Similarly, for this group of 28 RIS cases, the frequency of very large reservoirs is 6/28, of large reservoirs 11/28, and of small reservoirs 11/28. Were depth and volume unrelated, the frequency of reservoirs being both very deep and very large would be 10/28 times 11/28. Multiplying this times the number of reservoirs (i.e., 28) gives 3.93, the expected number of very deep and very large reservoirs given no correlation between the attributes. This number is shown in parentheses in Table 5 beside the original data.

To examine whether there is a correlation, the observed occurrences are compared to those predicted assuming independence. One way to do this is with a χ^2 goodness-of-fit test using the statistic

$$y = -2 \log \left\{ \frac{\prod_i n_i^{n_i} \prod_j n_j^{n_j}}{n^n \prod_i \prod_j n_{ij}^{n_{ij}}} \right\} \tag{1}$$

where n_{ij} is the number of occurrences in cell ij of the table, n_i and n_j are the number of occurrences along the row i and column j , respectively, and n is the total

TABLE 5
ILLUSTRATIVE TEST OF INDEPENDENCE BETWEEN TWO
ATTRIBUTES USING A CONTINGENCY TABLE*

		Volume			
		v_1	v_2	v_3	n_i
Depth	d_1	4 (2.14)	3 (3.93)	3 (3.93)	10
	d_2	7 (3.86)	8 (7.07)	3 (7.07)	18
n_j		6	11	11	$n = 128$

* The data in each cell is the actual number of reservoirs with the corresponding state description. The numbers in parentheses indicate the expected number of reservoirs assuming no correlation between depth and volume.

number of occurrences. Were the attributes independent, the statistic y would be distributed as a χ^2 distribution with $[(r - 1)(s - 1)]$ degrees of freedom, where r and s are the number of rows and columns, respectively (Kendall and Stuart, 1973). Thus, the observed value of this statistic can be compared with tables of χ^2 to determine its probability of exceedance for independent attributes. If the observed occurrences were very unlikely given independence, one would conclude that the attributes were in fact correlated.

For depth and volume, given RIS, the statistic y calculated from the data in Table 5 using (1) is 0.38. There are 2 degrees of freedom associated with this test. From a χ^2 table, one observes that χ^2 with 2 degrees of freedom is less than 5.99 ninety-five per cent of the time. Hence, it is not at all unlikely to obtain a statistic of 0.38 in this case. Consequently, using this discrete data, one can conclude that depth and volume are not strongly correlated for the RIS cases.

χ^2 statistics for each attribute pair for both the RIS and non-RIS sets are shown in Table 6. Associated degrees of freedom are shown in parentheses. Based on these analysis, the data do not support conclusions of dependence between any attribute

pair, given either RIS or non-RIS, with the possible exception of depth-volume. Even, this latter dependence is only weakly supported in the discrete data.

Unlike the other attributes, data on depth and volume for the 200 deep or very deep reservoirs were available as continuous variables. Normal regression analyses were performed to examine whether correlations between depth and volume were masked by the discrete assignments of Table 1. The results indicate weak correlations in both cases. The correlation coefficient between depth and the logarithm of volume for the RIS case was 0.07 and for the non-RIS case 0.22. Given the respective size of the data sets, 28 and 172, only the latter is significant at the 95 per cent level based on the *t* statistics reported in Table 6.

Relationship of microseismicity and macroseismicity at RIS sites. Attribute differences between sites which have had only microseismicity and those which have had macroseismicity were investigated using significance tests. Using the data

TABLE 6
TESTS OF INDEPENDENCE FOR ATTRIBUTE PAIRS

χ^2 Statistics for Discrete Attribute Combinations*						
	Depth	Volume	Stress	Faulting	Geology	
Depth		4.6 (2)	4.9 (4)	—	5.7 (4)	Non-RIS
Volume	0.38 (2)		5.7 (4)	—	4.5 (4)	
Stress	0.60 (4)	2.38 (4)		—	4.4 (4)	
Faulting	—	—	—		—	
Geology	2.7 (4)	0.63 (4)	1.8 (4)	—		
	RIS					
	RIS Case			Non-RIS Case		

Correlations of Depth and Volume from Regression Analysis on Continuous Data

0.07 for 28 reservoirs	0.22 for 172 reservoirs
$t_{28} = 0.36$	$t_{170} = 2.94$
$t_{0.95,28} = 1.706$	$t_{0.95,170} = 1.645$
Not significant at 95%	Significant at 95%

* Degrees of freedom associated with each test is indicated in parentheses beside the associated statistic. A statistic greater than 9.49 is significant at the 95 per cent level of confidence for 4 degrees of freedom, and a statistic greater than 5.99 is significant at that confidence level with 2 degrees of freedom

in Table 7, the test is based on the squared deviations from expected frequencies using the statistic

$$x = \sum_i \sum_j \sum_k \frac{(n_{ijk} - n_k p_i p_j)^2}{n_k p_i p_j}$$

which, assuming independence, has a χ^2 distribution with $2(r-1)(s-1)$ degrees of freedom where *r* and *s* are the numbers of rows and columns, respectively, n_{ijk} is the number of occurrences in cell *ij* of data *k* with *k* = 1 referring to the macroseismic data and *k* = 2 the microseismic data, n_k is the number in data table *k*, *n* is the total number in both tables, and

$$p_i = \sum_k \sum_j \frac{n_{ijk}}{n}$$

and

$$p_j = \sum_k \sum_i \frac{n_{ijk}}{n}$$

Resulting χ^2 statistics for the associated 8 degrees of freedom are shown adjacent to the data in Table 7. None of the differences are significant at the 90 per cent confidence level.

Shallow compared to deep and very deep RIS sites. Attributes associated with shallow sites reporting RIS and those associated with deep and very deep sites

TABLE 7
DATA AND STATISTICS TO COMPARE MICROSEISMICITY AND MACROSEISMICITY RIS RESERVOIRS*

Macroseismic RIS			Microseismic RIS			χ^2 Statistic†		
d_1	1	0	3	d_1	3	1	1	10.04
d_2	4	7	3	d_2	1	1	0	
d_3	5	2	1	d_3	3	2	0	
	v_1	v_2	v_3		v_2	v_2	v_1	
d_1	2	1	1	d_1	0	4	1	11.18
d_2	2	9	3	d_2	0	2	0	
d_3	2	4	2	d_3	1	3	1	
	s_1	s_2	s_3		s_1	s_2	s_3	
d_1	2	1	1	d_1	1	2	2	9.69
d_2	8	3	3	d_2	1	0	1	
d_3	2	4	2	d_3	2	1	2	
	g_1	g_2	g_3		g_1	g_2	g_3	
v_1	3	3	1	v_1	0	0	1	7.96
v_2	3	4	2	v_2	1	3	0	
v_3	0	7	3	v_3	0	6	1	
	s_2	s_2	s_3		s_1	s_2	s_3	
v_1	4	1	2	v_1	0	1	0	9.64
v_2	2	4	3	v_2	2	1	1	
v_3	6	3	1	v_3	2	1	4	
	g_1	g_2	g_3		g_1	g_2	g_3	
s_1	2	1	3	s_1	0	1	1	6.52
s_2	8	4	2	s_2	4	1	4	
s_3	2	3	1	s_3	0	1	0	
	g_1	g_2	g_3		g_1	g_2	g_3	

* Cell frequencies show numbers of reservoirs having paired combinations of attributes

† There are 8 degrees of freedom associated with each test. A statistic greater than 13.4 is significant at the 90 per cent level and greater than 15.5 is significant at the 95 per cent level.

reporting RIS were also examined for differences. Proceeding as above, χ^2 statistics for pairs of attributes involving volume, stress, and geology were calculated. Corresponding data tables for the various pairs of attributes are shown in Table 8 with χ^2 statistics and associate degrees of freedom. The only significant distinction between shallow RIS and deep/very deep RIS reservoirs was for the volume and stress pair of attributes. This was significant at the 95 per cent confidence level. These attributes may also be significantly different for non-RIS reservoirs, but data on shallow non-RIS reservoirs were not completely available.

4. A PRELIMINARY MODEL OF RIS

This section provides an initial model for assessing the probability of RIS given various attribute states. First, the probability of RIS, given the state of only one attribute, is examined to give a feeling for the association of different attribute states and RIS. This information is combined in a model, which assumes probabilistic independence of the attributes, to calculate the probability of RIS given states of all the attributes. Because of the correlation between depth and volume implied by analyses of Table 6, a second model was developed for estimating the probability of

TABLE 8
SHALLOW COMPARED TO DEEP AND VERY DEEP RIS RESERVOIRS*

Shallow Reservoirs (d_1)			Deep/Very Deep Reservoirs (d_1 or d_2)						
Relationship Between Attribute Pairs									
s_3	0	1	2	s_3	2	1	2	$\chi^2 = 10.55$	
s_2	4	2	1	s_2	8	3	5		$\chi^2_{0.90} = 13.4$
s_1	0	2	1	s_1	2	2	0		
	g_1	g_2	g_3		g_1	g_2	g_3		
v_3	3	3	2	v_3	5	1	3	$\chi^2 = 7.32$	
v_2	1	2	1	v_2	3	3	3		$\chi^2_{0.90} = 13.4$
v_1	0	0	1	v_1	4	2	1		
	g_1	g_2	g_3		g_1	g_2	g_3		
u_3	0	6	2	u_3	0	7	2	$\chi^2 = 20.28$	
u_2	3	1	0	u_2	1	6	2		$\sigma^2_{0.95} = 15.5$
u_1	0	0	1	u_1	3	3	1		
	s_1	s_2	s_3		s_1	s_2	s_3		
Single Attribute Differences									
s_1	3				4			$\chi^2 = 0.42$	
s_2	7				16				$\chi^2_{0.90} = 7.8$
s_3	3				5				
v_3	8				9			$\chi^2 = 3.44$	
v_2	4				9				$\chi^2_{0.90} = 7.8$
v_1	1				7				
g_1	4				12			$\chi^2 = 1.38$	
g_2	5				6				$\chi^2_{0.90} = 7.8$
g_3	4				7				

* Cell frequencies show numbers of attributes having paired attribute states.

RIS given this dependence. In particular, two specific cases were analyzed. One based on correlation between discrete depth and volume, and the other based on correlations between continuous depth and volume. The final subsection shows typical calculations of the probability of RIS using all three models: the independent model; the dependent-discrete model; and the dependent-mixed (discrete/continuous) model.

Single attribute model. Knowing the state of one attribute, e.g., depth d_1 , it is possible to calculate the conditional probability of RIS, denoted as $P(RIS | d_1)$, using

Bayes' theorem

$$P(RIS|d_i) = \frac{P(RIS)P(d_i|RIS)}{P(RIS)P(d_i|RIS) + P(\overline{RIS})P(d_i|\overline{RIS})} \tag{2}$$

where $P(RIS)$ and $P(\overline{RIS})$ are the prior probabilities of RIS and non-RIS, respectively, and $P(d_i|RIS)$ and $P(d_i|\overline{RIS})$ are the conditional frequencies (i.e., likelihoods) of depth d_i , given RIS and non-RIS, respectively.

For the 234 deep, very deep, or very large reservoirs, there are 29 cases of RIS. Thus, the prior probability $P(RIS)$ is $29/234 = 0.12$. Consequently, the probability of non-RIS is 0.88. Using this and the data of Table 2, the probability of RIS given any specific state of a single attribute can be calculated. To illustrate, for a very deep reservoir ($D = d_1$), equation (2) becomes

$$\begin{aligned} P(RIS|d_1) &= \frac{P(RIS)P(d_1|RIS)}{P(RIS)P(d_1|RIS) + P(\overline{RIS})P(d_1|\overline{RIS})} \\ &= \frac{(0.12)(0.34)}{(0.12)(0.34) + (0.88)(0.13)} \\ &= 0.26. \end{aligned} \tag{3}$$

TABLE 9
CONDITIONAL PROBABILITIES OF RIS GIVEN ONLY ONE ATTRIBUTE*

Attribute	State		
	1	2	3
Depth	0.27 (0.24)	0.11 (0.10)	0.03 (0)
Volume	0.12 (0.22)	0.23 (0.21)	0.09 (0.07)
Stress State	0.10 (0.11)	0.12 (0.14)	0.18 (0.17)
Fault Activity	0.18 (0.20)	0.0 (0.0)	—
Geology	0.16 (0.20)	0.10 (0.10)	0.12 (0.12)

* The numbers not in parentheses are based on the deep, very deep, and/or very large data set. Conditional probabilities in parentheses are based on deep or very deep data only.

The number 0.26 is referred to as the conditional probability of RIS, given that the reservoir is very deep. This and all the analogous conditional probabilities are shown in Table 9. From this information, it appears that the main attribute indicating whether a particular site is a RIS candidate is depth. [Faulting appears to be excellent for discrimination since the conditioned probability of RIS given no active faults in the vicinity is zero. However, because this is based on such a small set of data (seven RIS cases and six non-RIS cases), the result has little statistical meaning.] Volume is the information next most discriminating. Stress and geology attributes are not nearly such strong indicators, since within the data set, the conditional frequencies of these attributes given RIS and non-RIS are rather similar.

Multi-attribute model. Analogous to (2), considering all the attributes simultaneously,

$$P(RIS, |d, v, s, f, g) = \frac{P(RIS)P(d, v, s, f, g|RIS)}{P(RIS)P(d, v, s, f, g|RIS) + P(\overline{RIS})P(d, v, s, f, g|\overline{RIS})}; \tag{4}$$

and

$$P(\overline{RIS} | d, v, s, f, g) = \frac{P(\overline{RIS})P(d, v, s, f, g | \overline{RIS})}{P(RIS)P(d, v, s, f, g | RIS) + P(\overline{RIS})P(d, v, s, f, g | \overline{RIS})} \quad (5)$$

where $P(RIS | d, v, s, f, g)$ is the conditional probability of RIS given the combination of states d, v, s, f, g . Dividing (4) by (5) yields

$$\frac{P(RIS | d, v, s, f, g)}{P(\overline{RIS} | d, v, s, f, g)} = \frac{\left[\frac{P(RIS)}{P(\overline{RIS})} \right] \left[\frac{P(d, v, s, f, g | RIS)}{P(d, v, s, f, g | \overline{RIS})} \right]}{\frac{P(RIS)}{P(\overline{RIS})} LR(d, v, s, f, g)} \quad (6)$$

which in words says, "the conditional odds of RIS equals the prior odds of RIS multiplied by the likelihood ratio for the given states." Equations (4) and (6) are the bases of the models.

Preliminary model of RIS assuming probabilistic independence. Assuming probabilistic independence among all attributes, the conditional probabilities of (4) and (5) become

$$P(d, v, s, f, g | RIS) = P(d | RIS)P(v | RIS)P(s | RIS)P(f | RIS)P(g | RIS) \quad (7)$$

and

$$P(d, v, s, f, g | \overline{RIS}) = P(d | \overline{RIS})P(v | \overline{RIS})P(s | \overline{RIS})P(f | \overline{RIS})P(g | \overline{RIS}), \quad (8)$$

respectively. The terms $P(d | RIS)/P(d | \overline{RIS})$ are referred to as individual likelihood ratios. They are calculated from Table 2 and displayed in Table 10a. To use the independent model, one substitutes the information of Table 10, along with the prior probabilities of RIS and non-RIS, into (7) and (8) and then into either (4) or (6). Examples are included at the end of this section.

Obviously, using the two data sets of Table 10 yield different prior probabilities and likelihoods. However, the prior probabilities and likelihoods change such that the estimate of the probability of RIS using (7) and (8) is unchanged.

Models of RIS assuming dependence between depth and volume. A model very similar to that above holds even if probabilistic independence between volume and depth is not assumed. The result is that

$$P(d, v, s, f, g | RIS) = P(d, v | RIS)P(s | RIS)P(f | RIS)P(g | RIS) \quad (9)$$

and

$$P(d, v, s, f, g | \overline{RIS}) = P(d, v | \overline{RIS})P(s | \overline{RIS})P(f | \overline{RIS})P(g | \overline{RIS}) \quad (10)$$

where $P(d, v | RIS)$ means the joint probability that depth is d and volume is v , given that RIS occurred.

For the discrete case, this information can be estimated directly from the first data sets of Table 4, concerning depth and volume. For instance, for the 29 cases of RIS, three were both very deep and large (i.e., $D = d_1$ and $V = v_2$). Thus, the frequency of this combination given RIS is estimated as 3/29 or 0.11. In a similar manner, one can estimate the probability of a very deep and large reservoir given non-RIS to be 11/204 or 0.06. The likelihood ratio for the d_1, v_2 combination is the ratio of these numbers (Table 10b).

Equations (9) and (10) are useful for calculating the probability of RIS given discrete, although dependent, information on volume and depth. A similar model was developed treating (log) volume and depth as continuous variables. For the deep and very deep reservoirs, regression analyses were performed for both the RIS

TABLE 10
LIKELIHOOD RATIOS*

	State			
	1	2	3	
(a) Independent				
Depth	2.62 (2.26)	0.87 (0.76)	0.21	
Volume	0.95 (2.04)	2.15 (1.87)	0.66 (0.57)	
Stress	0.82 (0.74)	0.91 (1.0)	1.58 (1.29)	
Fault Activity	1.50 (1.50)	0.0 (0.0)	—	
Geology	1.34 (1.55)	0.74 (0.71)	0.94 (0.81)	
	Volume			
	Small	Large	Very Large	
(b) Dependent Discrete Case—Depth and Volume				
Depth	very deep	2.56 (2.22)	1.92 (1.67)	4.22 (3.66)
	depth deep	0.46 (0.40)	2.25 (1.95)	1.62 (1.41)
	shallow	—	—	0.21
(c) Dependent Continuous Case—Depth and Volume†				
$LR(d, v) = \frac{f_N(d, v \mu_d = 141, \mu_v = 3.21; \sigma_d = 48.8, \sigma_v = 1.0, \rho = 0.2)}{f_N(d, v \mu_d = 124, \mu_v = 2.78; \sigma_d = 26.8, \sigma_v = 0.88, \rho = 0.2)}$				

* The likelihood ratios in parentheses are based on deep and very deep reservoir data only.

† $f_N(\cdot, \cdot)$ indicates a normal distribution with parameters as given following the vertical sign.

and non-RIS cases to fit bivariate, normal distributions with dependence between the two attributes. Using these two distributions, one obtains the relative likelihoods of the occurrence of a particular d and v pair for the RIS and non-RIS cases. This likelihood ratio $LR(v, d)$ is given in Table 10c. One can use the likelihoods from the continuous data in model (6) to calculate the relative conditional probabilities (odds) of RIS to non-RIS for a particular reservoir. Because these probabilities must sum to 1, it is easy to calculate the conditional probability of RIS from this information.

Typical calculations for the probability of RIS. The likelihood equation (6) can be used to calculate the probability of RIS for all three cases: the independent case; the dependent-discrete case; and the dependent-mixed case.

To illustrate, consider the proposed Auburn dam, which if built, would be a very

deep, large reservoir in an extensional stress field, with active faulting present prior to impoundment, and metamorphic geology (d_1, v_2, s_1, f_1, g_2). For the independent model, data from Table 10a is substituted into equations (7) and (8) and then into (6) to calculate the probability of RIS as 0.35. Using the dependent-discrete model, data from Table 10, a and b, is substituted into equations (9) and (10) and then into (6) to find that the probability of RIS at Auburn as 0.17. Using the continuous model for volume and depth (the depth of Auburn reservoir is taken as 183 m and its volume $30 \times 10^9 \text{ m}^3$), the likelihood ratios of Table 10, a and c, are substituted into equation (6) to find the probability of RIS at Auburn to be 0.32. The basic data for all three calculations is shown in Table 11. These conditional probabilities are the same for both data sets as changes in the attribute likelihood are compensated by changes in the prior probabilities.

TABLE 11
AUBURN EXAMPLE*

States	Attributes		
	Likelihood Ratio		
	Independent Discrete	Dependent Discrete	Dependent Mixed, Discrete/Continuous
Very Deep	2.26	1.67	3.72
Large	1.87		
Extensional	0.74	0.74	0.74
Active	1.50	1.50	1.50
Metamorphic	0.71	0.71	0.71
$\Pi =$	3.33	1.32	2.93
Conditional Odds Ratio			
• Prior odds ratio = $0.14/0.86 = 0.16$			
• Conditional odds ratio			
independent = $0.16 \times 3.33 = 0.53$			
dependent discrete = $0.16 \times 1.32 = 0.21$			
dependent mixed = $0.16 \times 2.93 = 0.47$			
• Conditional probability of RIS			
independent = $0.53/1.53 = 0.35$			
dependent discrete = $0.21/1.21 = 0.17$			
dependent mixed = $0.47/1.47 = 0.32$			

* Based on deep and very deep data sets

There are several reasons for these answers to be different for the three models. First, based on the discrete data, there seems some indication of statistical dependence between volume and height. Therefore, some information about one is redundant with information about the other, conditional on RIS or non-RIS data. Consequently, treating them independently double counts this information, and a higher probability is calculated by using the independent model than by using the dependent-discrete model.

Second, when reservoir depth is categorized simply as deep or very deep, information is lost. In particular, from Figure 1, almost 60 per cent of the very deep reservoirs have experienced RIS. This is a strong indication that depth is related to RIS. Since Auburn is not only very deep but quite a bit deeper than the cut-off for very deep (i.e., it will be 183 m deep whereas the cut-off for very deep reservoirs is

150 m, the continuous model simply includes this information and hence predicts a higher likelihood of RIS.

To evaluate the precision of the estimate of 0.35 for the probability of RIS at Auburn reservoir, a sampling variance of the estimate was calculated (Packer *et al.*, 1979). To calculate this precision, the sampling variances of the marginal likelihoods \hat{p}_i , which is $\hat{p}_i(1 - \hat{p}_i)/n$, are propagated through Bayes' theorem using a Taylors series expansion. The corresponding standard deviation is 0.14, implying that the estimate is imprecise. Roughly and imprecisely, this means there is about a 68 per cent chance that the probability of RIS at Auburn dam is the range 0.34 ± 0.14 .

Table 12 gives the predicted likelihood for the occurrence of RIS at San Luis Dam and at dams which would be the most likely and least likely to induce seismicity, based on the discrete analyses and the current data set.

TABLE 12
SAMPLE CALCULATIONS*

Attributes	Likelihood Ratio					
	San Luis Reservoir (deep, large, extensional, active, sedimentary)		Best Case (deep, small, extensional, no activity, metamorphic)		Worst Case (very deep, very large, shear, active, sedimentary)	
	Independent	Discrete Dependent	Independent	Discrete Dependent	Independent	Discrete Dependent
Depth	0.76	1.96	0.76	0.40	2.26	3.66
Volume	1.87		0.57		2.04	
Stress	0.74	0.78	0.78	0.78	1.29	1.29
Faulting	1.50	1.50	†	†	1.50	1.50
Geology	1.55	1.55	0.71	0.71	1.55	1.55
$\Pi =$	2.45	3.55	0.24	0.22	13.8	11.0
Prior Odds Ratio	0.16	0.16	0.16	0.16	0.16	0.16
Conditional Odds Ratio	0.40	0.58	0.04	0.04	2.26	1.80
Conditional Probability of RIS	0.29	0.37	0.04	0.03	0.69	0.64

* Shallow reservoirs not included.

† Empty data cell with likelihood ratio zero. Because this is based on a very small data set, we assume the likelihood ratio to be 1.0 in this calculation.

5. INTERPRETATION OF THE RESULTS

Several conclusions can be tentatively drawn from the results. First, from Table 9, it can be seen that of the five attributes, depth is the one which best discriminates circumstances which may or may not lead to RIS. For very deep reservoirs, the conditional probability of RIS is 0.27 and for shallow reservoirs it is 0.03. This range is larger than for any other attribute. In interpreting this range, one should recognize that the data set includes only deep, very deep, and/or very large reservoirs. Shallow reservoirs that are not very large are not included in the analysis, and would of course have a probability of RIS very near zero.

The next best attribute for distinguishing between RIS and non-RIS cases is volume of the reservoir. Again, the present numerical results exclude shallow reservoirs that are not very large. Were these included, both the likelihood ratios and prior probabilities of RIS would compensatingly change.

From Table 9, it can be seen that the conditional probability of RIS, given a large reservoir, is 0.23, and the conditional probability of RIS, given a very large reservoir, is 0.12. This seems inverted, but is merely an anomaly caused by including shallow but very large reservoirs in the data set. There are 33 such dams for which there has been no RIS and only one where there has been RIS. If one excludes these dams from the data set, there are 18 very large dams without RIS and 6 with RIS. Similarly, there are 36 large dams without RIS and 11 with RIS. Thus, the conditional probability of RIS for a very large reservoir is 0.22, whereas for a large reservoir it is still 0.21.

Based on the current data base the set of attributes, taking one attribute at a time, most conducive to RIS is the following: a very deep, very large reservoir in a shear stress zone with active faulting present prior to the reservoir's existence and in sedimentary formations (Table 12).

Because there are so few data on the presence of active faulting prior to the existence of the reservoir, little can be concluded statistically about the relevance of this attribute. It is difficult to ascertain that there are no active faults in an area. However, if it were known for certain that no active faults existed in an area, the probability of RIS would be reduced.

Examining Table 6, there seem no strong correlations among attributes conditioned on the occurrence or on the nonoccurrence of RIS. The one exception is possible correlation between depth and volume. This correlation, although weak, is substantiated by the analysis of continuous data on depth and volume for non-RIS sites.

The continuous analysis indicated the apparently strong relationship between the likelihood of RIS and depth of the reservoir. One can almost see this relationship simply by examining Figure 1. As a result of that relationship, the dependent-mixed model seems the best of the three predictive models. It includes dependency between depth and volume and uses the data as continuous variables. Of course, it is wholly inappropriate to conclude that any of these models is "the" correct model.

From Table 7, there is no reason to conclude that the characteristics of reservoirs experiencing microseismic and macroseismic events are distinguishable from each other with regard to the five attributes in the analysis. Table 8 indicates essentially no significant differences were found between shallow and deep/very deep reservoirs experiencing RIS. Both of these conclusions, however, are based on a small data set (38 cases of RIS).

A last, perhaps self-evident observation is that cause and effect relationships are not implied by these analyses. For instance, higher pore pressures may be a real cause of triggering RIS. Certain characteristics of geology and the reservoir are simply more conducive for the build-up of pore pressure. It may also be that the present attributes and RIS are correlated to yet unidentified causal attributes.

Appraisal of the data. Several comments about the data set are important to understanding the results. In developing any data set, it was necessary to make assumptions (see Packer *et al.*, 1979). In all cases, there was an attempt to make these professional judgments in a systematic and justifiable manner.

The first judgment was in selecting attributes to characterize the reservoirs. This was based on the assumed correlation of the attributes with RIS and on data availability. Other attributes might have been included (e.g., water level fluctuation, pore pressure changes), but only for the difficulty of collecting such data.

The second judgment was in classifying reservoir sites. For some sites, detailed information on local geology and stress is not available; consequently, geology and

stress were presumed from regional information. Even depth and volume data may in certain cases be inaccurate. However, errors of classification would not be thought systematically biased.

The third judgment was in discrete classification. In particular, the conclusions can be changed somewhat by choosing the volume and depth cut-offs differently. For instance, from Figure 1 it can be seen that several reservoirs slightly over 150 m had no RIS. Changing the cut-off for very deep reservoirs to 175 m from 150 changes the conditional probability of RIS for very deep reservoirs to 0.40 from 0.27. Of course, "very deep" now has a different meaning. The continuous variable model circumvents this problem.

The final major judgment was in deciding whether particular reservoirs were associated with RIS. This was based on available seismic records in the vicinities of the sites. An attempt was made to identify temporal and spatial associations of seismic activity with the filling, drawdown, and refilling of the reservoirs. However, in many cases, there were no seismic records prior to the reservoir's construction, and no local seismic net to detect minor events. As with any statistical analysis, results depend on current understanding of the historical record. As that understanding changes, a reassignment of RIS cases may occur with corresponding changes in the implications of the data.

For interpreting the present results, or any model of RIS, it is important to maintain perspective. There is no method or methodology, nor could there be one, which is completely objective for developing a model of the likelihood of RIS. Professional judgments are always necessary. The ultimate goal of such models is to clarify these professional judgments and to provide a basis for communication, modification, and improvement to better understand the phenomenon of RIS.

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